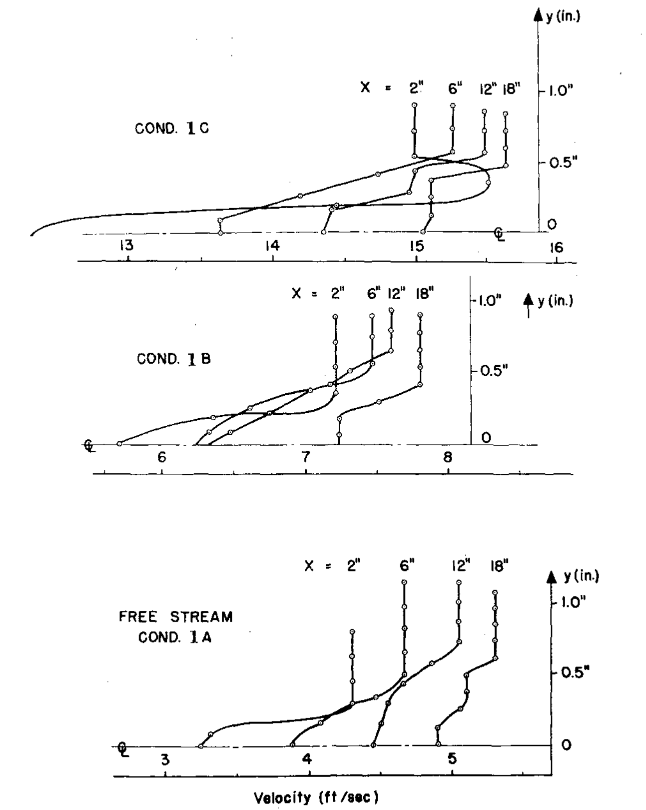

Fig. 1 Velocity profiles measured for three freestream velocities

The formation of several nearly discrete cylindrical shear regions is evident in Fig. 1. Under certain conditions of high speed, high angle of attack, and small wing-tip separation, as many as four such layers were obtained, as shown in Fig. 2.

The data shown in Fig. 2 could not be repeated from day to day, which may indicate a strong sensitivity of the phenomena to test parameters. Excellent qualitative consistency was observed, however, in all tests. The uncertainties and inaccuracies of the experiment were much too small to alter the shape of the velocity profiles.

Conclusions

A vortex wake has a tendency to form rather sharp piecewise cylindrical shear layers, which may or may not be unstable, depending on swirl velocity, stream velocity, and swirl profile. Similar phenomena can be observed in condensation trails from airplanes.

The phenomenon is related to similar phenomena occurring in flows in rotating containers. The region near the center of a vortex rotates almost as a solid body, whereas in the outer region the flow approaches a potential vortex flow.

The velocity field in the vortex core therefore may be governed by the effects of angular velocity, for example, as a possible tendency for the velocity field to be aligned with the vortex lines with a resulting modification of the flow stability. The data suggest that vortex bursting may not be a single phenomenon, but rather a sequency of quite orderly processes.

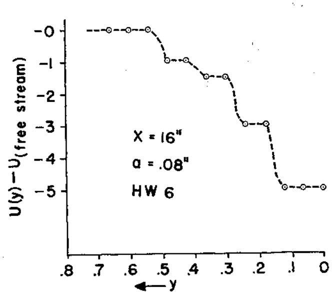

Fig. 2 Velocity profile showing four plateaus. More points than indicated were taken

Linearized Interaction Curves for Plastic Beams under Combined Bending and Twisting

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It has been shown by Handelman[1] and by Hill[2] that for a prismatic beam of perfectly plastic material, subjected to combined bending and torsion, the plastic-stress function $\phi(x, y)$ at a given section can be determined by solving the differential equation

$$\frac{\partial}{\partial x}\left[\frac{y\phi_x}{(1-\phi_x^2-\phi_y^2)^{1/2}}\right]+\frac{\partial}{\partial y}\left[\frac{y\phi_y}{(1-\phi_x^2-\phi_y^2)^{1/2}}\right]\frac{\mu}{\lambda}=0 \tag{1}$$

together with the condition $\phi_s = 0$ along the boundary. In Eq. (1), μ represents the ratio of the rate of twist to the rate of curvature, and λ is a constant depending on the yield condition assumed in the analysis. When the stress function ϕ throughout the section is known, the limiting combination of the bending moment M and the twisting moment T may be computed by the following expressions:

$$M = \sigma_0 \int\int y(1-\phi_x^2-\phi_y^2)^{1/2}dxdy \tag{2}$$

$$T = 2\tau_0 \int\int \phi dxdy \tag{3}$$

in which σ_0 and τ_0 are the yield stresses in simple tension and in shear, respectively.

Equation (1) is a nonlinear partial differential equation and can be solved numerically by Southwell's relaxation method. Recently Steel[3] and Imegwu[4] have obtained a number of solutions for circular, square, and triangular cross sections. Their results are summarized in a nondimensional form in Fig. 1. It is interesting to note that the plastic interaction between bending and torsion is virtually independent of the cross-sectional shape of the beam.

In solving practical problems, it is often convenient to use the so-called piecewise linear interaction curves that are derived from the true interaction curve through proper linearization. Obviously, the accuracy of the solution to a given problem depends directly on how close the linear interaction curves approximate the actual curve. Because of the lack of an exact solution, Sankaranarayanan and Hodge[5] have suggested a two-segment linear approximation based on a lower-bound interaction curve. Unfortunately, this approximation, shown as the dotted line in Fig. 1, deviates appreciably from the numerical results obtained by Steele and Imegwu. A new type of linear approximation therefore is proposed herein. It consists of four linear segments AB,

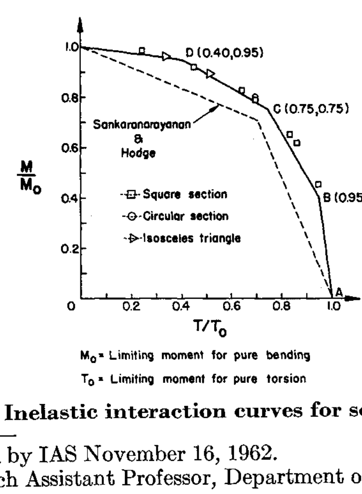

Fig. 1 Inelastic interaction curves for solid sections

Received by IAS November 16, 1962.

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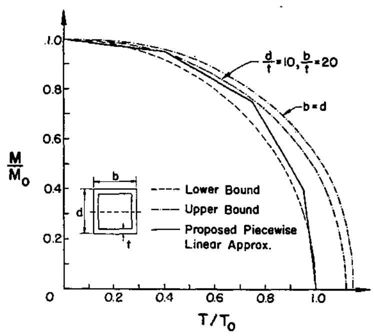

Fig. 2 Inelastic interaction curves for thin-walled box sections

BC, CD, and DE passing closely through the theoretically computed points. The flow rule and the energy-dissipation rate associated with this approximation can be determined easily by considering the normality condition of the strain-rate vector.

The new interaction curve is also valid for thin-walled box beams. Figure 2 shows a comparison of this curve with the upper- and lower-bound solutions obtained by Gaydon and Nuttall.[6] The proposed interaction curve closely approximates the lower-bound solution and is everywhere within the upper bounds.

References

[1] Handelman, G. H., "A variational principle for a state of combined plastic stress," Quart Appl. Math. **1**, 351–353 (1944).

[2] Hill, R., "A variational principle of maximum plastic work in classical plasticity," Quart. J. Mech. Appl. Math. **1**, 18–28 (1948).

[3] Steele, M. C., "The plastic bending and twisting of square section members," J. Mech. Phys. Solids **3**, 156–166 (1954).

[4] Imegwu, E. O., "Plastic flexure and torsion," J. Mech. Phys. Solids **8**, 141–146 (1960).

[5] Sankaranarayanan, R. and Hodge, P. G., Jr., "On the use of linearized yield conditions for combined stresses in beams," J. Mech. Phys. Solids **7**, 22–36 (1958).

[6] Gaydon, F. A. and Nuttall, H., "On the combined bending and twisting of beams of various sections," J. Mech. Phys. Solids **6**, 17–26 (1957).

Scaling of Jet Flameholders

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The problem of scaling when the flame stabilizer is a gasjet is considered. This study requires that the value of certain factors such as inlet temperatures and equivalence ratios be equal in model and prototype if scaling is to be considered. As is usually the case, some dimensionless parameters must be allowed to vary during scaling. The requirement that the mainstream-jet interaction be retained during scaling is used in selecting the mainstream Mach number rather than Reynolds number as a scaling parameter. The variation of these two factors during scaling is also considered in reaching this decision. The other scaling parameters are the pressure ratio across the jet and the ratio of residence time to reaction time in the combustor.

Nomenclature

A = area
c = constant
F = stoichiometric fuel-air ratio
L = characteristic dimension
$\dot{m}$ = mass rate of flow
M = Mach number
n = effective overall order of reaction
P = static pressure
q = dynamic pressure
Re = Reynolds number
T = temperature
V = velocity
w = rate of reaction per unit volume
η = combustion efficiency
ρ = density
τ = dimensionless time ratio
φ = equivalence ratio

Subscripts

c = critical zone
cj = into critical zone from jet
cs = into critical zone from mainstream
j = jet
m = model
p = prototype
s = mainstream

A STREAM of air or air-fuel mixture injected upstream in a flow of combustible gas has been found[1] to have the ability to stabilize a flame in this flow. Such a flame-stabilizing mechanism is commonly called a jet flameholder to contrast it to the more common bluff-body type.

Studies of the jet flameholder have been largely experimental because of the complexity of the flameholding mechanism and the large number of variables involved.

This note is an effort to assist in the use of the jet flameholder through a study of similitude parameters to allow scaling from a successful test model to a full scale prototype. A knowledge of scaling is desirable, since both turbojet afterburners and ramjet combustion chambers, where flameholders are used, can be large in size.

The combustion efficiency will be investigated as a function of the various dimensionless parameters. Other combustion chamber dimensionless dependent variables, such as $\Delta T/T_s$ or $\Delta P/P_s$, also are covered by the discussion, as they are functions of the same dimensionless parameters as is η.

Under the following conditions (which apply throughout this note): 1) fixed fuel, 2) fixed equivalence ratios in mainstream and jet, 3) fixed mainstream and jet temperatures, and 4) fixed fuel temperatures, the combustion efficiency for a group of geometrically similar arrangements can be written as

$$\eta = f_1(M_s, M_j, Re_s, Re_j, \tau) \quad (1)$$

Way[2] has suggested a method of handling the dimensionless time ratio τ. He suggests writing the rate of reaction in the form of an effective overall nth order reaction.

$$w = c\rho^n \quad (2)$$

Equation (2) is valid for constant inlet temperatures and fuel composition providing that the usual complicated chain reaction kinetics can be described accurately by a single nth order reaction.

The dimensionless time ratio, which is residence time divided by reaction time, is written as

$$\tau = \left(\frac{L/V}{\rho/w}\right)_s = \left(\frac{Lw}{\rho V}\right)_s = \left(\frac{cL\rho^{n-1}}{V}\right)_s \quad (3)$$

In the case of a jet flameholder, the factors used in Eq. (3) are those of the mainstream, since the mainstream conditions are more important provided that the interaction between jet and mainstream is fixed. There are thus three similitude parameters for the mainstream in Eq. (1). As pointed out in Ref. 2, setting the value of these three parameters equal in

Received by ARS November 5, 1962.
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